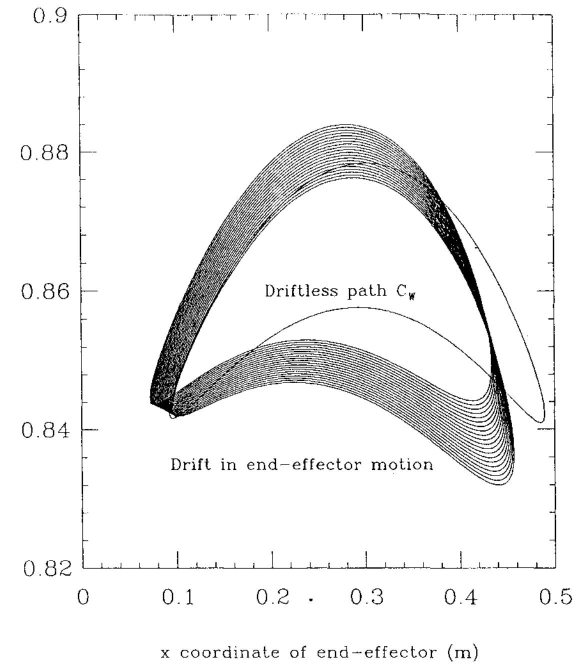

Fig. 3 Path I in the joint space, shown in Fig. 2, fails to produce a closed path in the workspace of the space manipulator; path II produces repeatable motion.

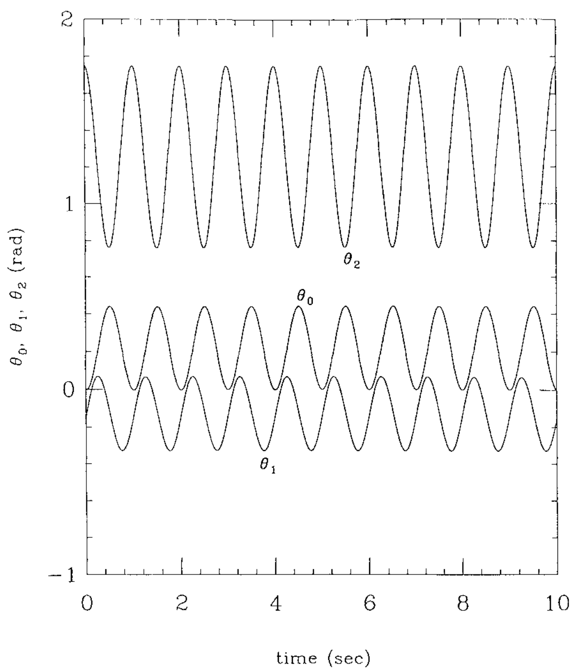

Fig. 4 Evolution of the orientation of the space vehicle and the joint angles of the space manipulator as the manipulator tracks the closed end-effector trajectory in Fig. 3 for 10 cycles using pseudoinverse control of the generalized Jacobian.

This closed path is $\mathcal{C}_\mathcal{W}$. Figure 4 shows the evolution of θ_0, θ_1, and θ_2 as the end effector traces the closed path $\mathcal{C}_\mathcal{W}$ in the workspace. It can be seen that the drift in the configuration variables is negligible.

VI. Conclusions

We discussed the repeatability problem in free-flying planar space robots. We showed that there exist certain closed trajectories in the joint space as well as in the workspace along which the space robot can exhibit holonomic behavior globally. We addressed the motion planning problem that relies on exact knowledge of the system parameters. Future research will be directed toward developing feedback control laws for repeatable motion in planar space manipulators as well as manipulators in three dimensions.

Acknowledgment

This work was funded in part by National Science Foundation Grant CMS-9410157.

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Prescribed Pole Placement with Optimal Weight Selection for Single-Input Controllable Systems

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Introduction

THE linear quadratic regulator (LQR) has been proven to be a robust design.[1] The combination of the method of LQR and pole placement has been intensively studied in recent decades.[2–5] Reference 2 showed that standard single-input controllable systems could be optimized by solving a Lyapunov equation instead of the matrix Riccati equation. A method for optimally moving the imaginary parts of the open-loop poles to a horizontal strip was shown in Ref. 3. State feedback control law was used to assign all closed-loop poles to a specified disk centered on the negative real axis in

Received April 15, 1995; revision received Aug. 7, 1995; accepted for publication Aug. 14, 1995.

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Ref. 4. Reference 5 presented yet another method for placing all closed-loop poles within a vertical strip.

Reference 6 indicated that in certain cases negative state weight and nondefinite state weight matrices are needed to place the closed-loop poles in the desired area. References 7 and 8 showed that the negative state weight matrix is needed for obtaining satisfactory aircraft flight quality. A closed-loop aircraft system with acceptable performance can be achieved only through very careful selection of negative and nondefinite state weight matrices. Therefore, it is desirable to develop a method for placing the closed-loop poles exactly at prescribed points and for finding the corresponding state weight matrix.

Saif[9] presented a recursive algorithm for selecting the state weight matrix of an LQR problem based on specified closed-loop poles. The method, however, cannot be used for repeated roots and the Hamiltonian matrix has to be solved several times to find the desired state weight matrix. Reference 10 also presented a recursive method for selecting the state weight in order to shift the poles to the desired locations. This method, however, requires decomposing the original system into a combination of several second-order systems and a first-order system with one nonlinear algebraic Riccati equation (ARE) for each subsystem. The total number of these AREs depends on the order of the initial system. Both techniques can be computationally challenging for real-time control of fast moving time-varying systems.

The purpose of the current paper is to present a method for exact placement of the poles of single-input controllable systems. It is shown here that obtaining the optimal state weight matrix for such cases is simple and can be easily programmed.

Method of Analysis

Consider a single-input time-invariant controllable system

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}, \qquad \text{and} \qquad \hat{x}(0) = \hat{x}_0 \tag{1}$$

where $\hat{x}$, $\hat{u}$, $\hat{A}$, and $\hat{B}$ have appropriate dimensions. It is assumed here that a time-varying system can be modeled by a series of time-invariant systems over discrete time intervals. Because the system is controllable, the controllability matrix, $\hat{P}_c$ defined next is invertible,

$$\hat{P}_c = [\hat{B} \quad \hat{A}\hat{B} \quad \hat{A}^2\hat{B} \quad \cdots \quad \hat{A}^{n-1}\hat{B}] \tag{2}$$

Letting $\hat{x} = Tx$, where T represents a transformation matrix, the preceding system can be expressed in the following second canonical form:

$$\dot{x} = Ax + Bu, \qquad \text{and} \qquad x(0) = x_0 = T^{-1}\hat{x}(0) \tag{3}$$

The state weight matrix of the preceding system, A, and the control matrix B are now

$$A = T^{-1}\hat{A}T = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix} \tag{4}$$

$$B = T^{-1}\hat{B}T = [0 \quad \cdots \quad 0 \quad 1]^T \tag{5}$$

The invertible controllability matrix of this second canonical system is

$$P_c = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B] \tag{6}$$

Substituting Eq. (4) into Eq. (6), and rearranging the position of the two controllability matrices, the transformation matrix can, therefore, be obtained as follows:

$$T = \hat{P}_c P_c^{-1} \tag{7}$$

Assume the performance index for the LQR for the transformed system is given by

$$J = \frac{1}{2}\int_0^\infty [x^T Q x + u^T R u]\, \mathrm{d}t \tag{8}$$

where Q is diagonal and R is the identity matrix. The choice of Q is arbitrary, but only a diagonal form will result in a unique solution. Under these assumptions, the optimal control is given by

$$u = -R^{-1}B^T P x \tag{9}$$

where the closed-loop Lyapunov matrix P satisfies the ARE

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{10}$$

The closed-loop state matrix is given as

$$A_c = A - BR^{-1}B^T P \tag{11}$$

Using the given forms of B and R, one can easily show that the last term of Eq. (11) reduces to the following special form:

$$BR^{-1}B^T P = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 \\ P_{n,1} & P_{n,2} & \cdots & P_{n,n} \end{bmatrix} \tag{12}$$

Substituting this into Eq. (11) shows that the closed-loop state matrix is given by

$$A_c = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_0 - P_{n,1} & -a_1 - P_{n,2} & \cdots & -a_{n-1} - P_{n,n} \end{bmatrix} \tag{13}$$

The characteristic equation of the closed-loop system is now

$$\prod_{i=1}^{n}(s - \lambda_i) = s^n + a_{c_{n-1}}s^{n-1} + \cdots + a_{c_1}s + a_{c_0} = 0 \tag{14}$$

where λ_i represents the ith eigenvalue of the closed loop system. Because these eigenvalues are prescribed, the coefficients of this polynomial are known. Therefore, the elements of the last column of the closed-loop Lyapunov matrix P can be determined by comparing Eqs. (13) and (14) or

$$\begin{cases} P_{n,1} = -a_0 + a_{c_0} \\ P_{n,1} = -a_1 + a_{c_1} \\ \quad\vdots \\ P_{n,n} = -a_{n-1} + a_{c_{n-1}} \end{cases} \tag{15}$$

The nonlinear term of the ARE, that is, $(PBR^{-1}B^T P)$, involves only the elements of the last column of the closed-loop Lyapunov matrix. Knowing these elements removes the nonlinearity of this equation. The ARE offers $n(n+1)/2$ linear equations for determining the closed-loop Lyapunov matrix. The unknowns in these equations are the elements of Q, which is a diagonal matrix, and all of the elements of P except for those of the last column.

To transform the second canonical system back into the original system, the original closed-loop Lyapunov matrix can be simply found from the closed-loop state matrix as follows:

$$\hat{A}_c = \hat{A} - \hat{B}R^{-1}\hat{B}^T\hat{P} = T(A - BR^{-1}B^T P)T^{-1} \tag{16}$$

Therefore, the closed-loop Lyapunov matrix of the original system is

$$\hat{P} = T^{-T} P T^{-1} \tag{17}$$

Also, substituting $x = T^{-1}\hat{x}$ into Eq. (8) results in

$$\hat{Q} = T^{-T} Q T^{-1} \tag{18}$$

The method is outlined in detail in Ref. 11, along with a numerical example.

Results and Discussion

Numerical Example

Longitudinal motion of an aircraft from Ref. 12 is modeled as

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}$$

$$= \begin{bmatrix} -0.0589 & 11.3448 & 0 & -32.1581 \\ -0.0022 & -0.6022 & 0.9786 & -0.0059 \\ -0.0002 & -1.8583 & -1.1776 & 0.0019 \\ 0 & 0 & 1 & 0 \end{bmatrix} \hat{x}$$

$$+ \begin{bmatrix} 0 \\ -0.0455 \\ -3.0313 \\ 0 \end{bmatrix} \hat{u} \tag{19}$$

where the state vector $\hat{x} = (V_x, V_z, q, \theta)^T$ and the control $\hat{u} = \delta_e$, the elevator deflection angle. Assume it is desired to place the poles at $(-2, 0)$, $(-2, 0)$ for the short period mode and $(-0.5, 0.5)$, $(-0.5, -0.5)$ for the phugoid mode. These values have been chosen quite arbitrarily. In practice, however, these eigenvalues will be determined based on some rationale to achieve a desired system behavior. Applying the method, one can show that the closed-loop state matrix of the original system becomes

$$\hat{A}_c = \hat{A} - \hat{B}R^{-1}\hat{B}^T\hat{P}$$

$$= \begin{bmatrix} -0.0589 & 11.3448 & 0 & -32.1581 \\ -0.0008 & -0.5675 & 0.9306 & -0.0981 \\ 0.0902 & 0.4520 & -4.3736 & -6.1379 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{20}$$

The intermediate steps of the calculations have not been shown in the interest of brevity. The reader is referred to Ref. 11 for further details.

The closed-loop response is compared with that of the open-loop system in Fig. 1 for the pitch angle. This figure clearly shows the original system oscillates for a long time before reaching the steady state. However, the closed-loop system based on the optimal pole placement method converges in approximately 10 s. The time history of the control input in this case, which is not shown, indicates that the magnitude and the rate of the elevator deflection required for control is quite reasonable and can be easily attained. The initial conditions used for the above example were

$$\hat{x}(0) = [0\,\text{ft/s},\ 0\,\text{ft/s},\ 5\,\text{deg/s},\ 0\,\text{deg}] \tag{21}$$

Discussion and Comparisons

The canonical controllable single-input state equation is frequently used in optimal control design. In those cases where the closed-loop poles are unreachable,[6] the ARE will result in a solution only through the eigenvector approach. For example, when the state weight matrix is negative definite or nondefinite, the ARE solution is not positive definite anymore. The method outlined here, however, allows for the solution of the inverse problem; that is, the state weight matrix can be found given the desired closed-loop poles. In fact, all controllable single-input systems can be transformed into the second canonical form. This implies that the single-input optimal pole placement design becomes a linear algebraic problem.

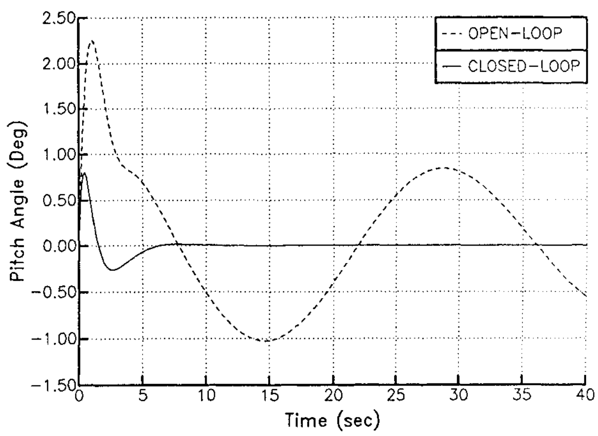

Fig. 1 Time history of pitch angle.

To show that this method is more powerful and effective than other methods presented in Refs. 3–5, 9, and 10, the following comparisons are made. The reader is reminded that many of the following methods are not applicable to aircraft models where the system eigenvalues can be different by several orders of magnitude.

Comparison with the Method of Reference 3

Reference 3 presented a method for shifting the imaginary parts of poles. This technique also affects the real parts of the closed-loop eigenvalues. This method requires a special transformation matrix to decompose the original system into several subsystems. The nature of the transformation does not render itself easily to programming. Additionally, cases involving more than two inputs appear to result in noncausal second-order subsystems. This issue is not addressed in Ref. 3. Furthermore, the effect of this method on an unstable open-loop system is unclear. The current method allows exact placement of the closed-loop poles with minimal computational effort.

Comparison with the Method of Reference 4

Reference 4 presented a method to shift the closed-loop pole to a specified disk centered on the negative real axis. If it is necessary for the imaginary parts of the closed-loop poles to be very large, then this method will fail to achieve its objective. In fact, if the real parts of the closed-loop eigenvalues are different by as much as one order of magnitude, then this method will fail. The current method does not suffer from this weakness.

Comparison with the Method of Reference 5

Reference 5 presented a recursive method for shifting the open-loop poles to a vertical strip. Like Ref. 3, this method requires a special transformation matrix to separate the original system into several subsystems. In this method, the reduced ARE has to be solved once for each subsystem. This can be computationally intensive for real-time control of fast moving time-varying systems. Another problem that arises from using this method for control of aircraft is the relative magnitudes of the system eigenvalues. To accommodate both sets of eigenvalues, this strip has to be quite wide. Furthermore, exact placement of the poles within the strip is not possible. Therefore, it is conceivable for the transformation to result in closed-loop poles that coincide.

Comparison with the Methods of References 9 and 10

References 9 and 10 presented techniques for finding the desired state weight for prescribed eigenvalues. The method of Ref. 9, however, cannot be used for repeated roots, and finding the desired state weight requires the Riccati equation to be solved repeatedly. Also, the state weight matrix is not unique, which can cause computational difficulties. The approach of Ref. 10 is similar to that of Refs. 3 and 5. This method requires a special transformation matrix to decompose the original system into several subsystems. The effect of the transformation on the control matrix is not explained clearly in Ref. 10, however, especially for cases with more than two control inputs. In the current method, the solution can be obtained from a set of linear algebraic equations, and the transformation matrix can be found quite simply.

Although the current method has the stated advantages, it suffers several limitations as well. One is that this method is limited to single-input controllable systems. If the system is stabilizable, then uncontrollable poles must be identified and the transformation matrix has to be modified. Also, this approach can be used with multi-input systems, only if they can be controlled by one input. Furthermore, this method requires the availability of the full state vector. Otherwise, it has to be employed along with an estimator to provide an estimate of this vector.

Conclusions

A method was presented for exact pole placement, using optimal state weight selection, for single-input controllable systems. It was shown that the difference between the open-loop and closed-loop characteristic equations can be used to determine the elements of the last column of the closed-loop Lyapunov matrix. These terms change the nonlinear portion of the algebraic Riccati equation to known values so that the elements of the desired state weight matrix, as well as the remaining terms of the closed-loop Lyapunov matrix, can be determined from a set of linear algebraic equations. The linearized longitudinal equations of motion of an aircraft were used to demonstrate the application of the outlined procedure.

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Approximate Pole Placement for Acceleration Feedback Control of Flexible Structures

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Introduction

THE vibration control of flexible structures is a tricky problem because a finite order controller is used to control an (essentially) infinite dimensional system. Robustness is of paramount importance here because an accurate knowledge of the system is often lacking, and it is impossible to model all of the modes. Furthermore, there are several destabilizing factors, such as the effect of spillover[1] and the interaction of finite bandwidth actuator's dynamics,[2] all of which must be carefully considered when designing the controller. In Ref. 3, an active vibration control technique for second-order systems using acceleration feedback was proposed and shown to be unconditionally stable. Its robustness is further demonstrated in actual experiments[4] where single-input/single-output (SISO) controllers are designed to augment the damping of a single target mode. In a more restricted context, Sim and Lee[5] also show that after incorporating finite actuator dynamics the collocated acceleration feedback control scheme is globally and unconditionally stable. No constructive procedure, however, is proposed therein to assign desired closed-loop damping to the controlled modes.

This Note proposes a procedure for the design of a multi-input/multi-output (MIMO) controller that assigns prescribed closed-loop damping to multiple target controlled modes. We show that up to 50% closed-loop damping can be assigned to each of the controlled modes approximately, while at the same time ensuring that all the uncontrolled and unmodeled modes remain stable with higher than natural closed-loop damping.

Acceleration Feedback of a Scalar Second-Order System

To understand how the pole-placement procedure works, we first examine the acceleration feedback control of a scalar second-order system. As in the case of positive position feedback control,[2] we associate a tuning filter with sufficiently high open-loop damping to the system, and transfer some of the open-loop damping of the tuning filter to the system via an appropriate arrangement. Consider the following closed-loop arrangement of a scalar second-order system with a second-order tuning filter.

System:

$$\ddot{y} + 2\zeta_n\omega_n\dot{y} + \omega_n^2 y = -\gamma^2\eta \tag{1}$$

Tuning filter:

$$\ddot{\eta} + 2\zeta_f\omega_f\dot{\eta} + \omega_f^2\eta = \omega_f^2\ddot{y} \tag{2}$$

where $\omega_n > 0, \zeta_n > 0, \omega_f > 0$, and $\zeta_f > 0$ are the natural frequency and natural damping ratio of the system and natural frequency and natural damping ratio of the filter, respectively. Here, γ^2 is a scalar gain to be determined.

Proposition 1. The closed-loop system (1) and (2) is unconditionally stable.

Proof. This follows readily by a direct application of the Routh–Hurwitz criterion on the closed-loop characteristic equation

$$P(s) = \left(s^2 + 2\zeta_n\omega_n s + \omega_n^2\right)\left(s^2 + 2\zeta_f\omega_f s + \omega_n^2\right) + \gamma^2\omega_f^2 s^2 = 0 \tag{3}$$

As a design problem, the filter's parameters and the gain are to be determined such that a desired closed-loop damping for the system is to be achieved. We assume that the natural damping of the system is very small ($\zeta_n \ll 1$) (otherwise there is no need for active damping enhancement), hence the system's open-loop poles are close to the imaginary axis. A quick root-locus reveals that the most effective tuning filter's parameters are given by $\zeta_f = 1$ (critically damped filter) and $\omega_f = \omega_n$. The corresponding root locus diagram is as shown in Fig. 1, where ζ_n is assumed to be zero and ω_n is normalized to unity. Maximal closed-loop damping for the system is achieved at the breakaway point where the branch emanating from the system's open-loop pole meets the branch emanating from the filter's open-loop pole. The breakaway point occurs when $\gamma^2 = 1$ regardless of the value of ω_n. Since under the assumption that $\zeta_n = 0, \omega_n = \omega_f, \zeta_f = 1$, the characteristic equation reduces to

$$\left(s^2 + \omega_n^2\right)(s + \omega_n)^2 + \omega_n^2 s^2 = \left\{[s + (\omega_n/2)]^2 + \left(3\omega_n^2/4\right)\right\}^2 = 0 \tag{4}$$

hence the repeated poles are to be found at $s = -\frac{1}{2}\omega_n \pm \sqrt{(3/2)}\omega_n i$ resulting in a maximal closed-loop damping ratio of 50% for both the system and the tuning filter. Increasing γ^2 beyond unity will not increase the damping of the system or the filter; but nevertheless it will not destabilize the system either.

Once the filter parameters are tuned to the system's, it is easy to design the feedback gain to achieve a prescribed closed-loop

Received June 17, 1994; revision received May 7, 1995; accepted for publication Sept. 13, 1995.

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